



higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA



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**Knowledge and Practice Standards
Number Sense WG
Corin Mathews, Lise Westeway, Zain Davis**



Knowledge and Practice Standards (KPS) 9 to 17

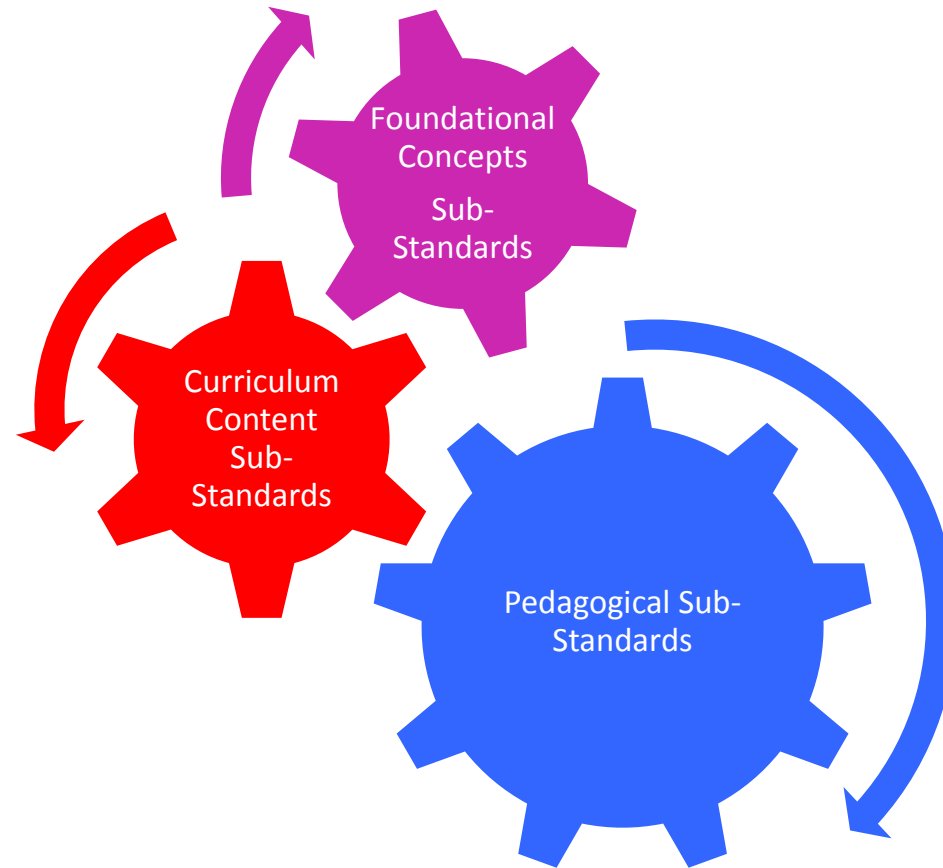


Knowledge and Practice Standard 12: Additive Relations with whole numbers: *Understand counting as a fundamental mediating resource used to relate operations on aggregates to the basic arithmetic operations. Understand the use of additive relations in the teaching of addition and subtraction of natural numbers. Teachers should know addition and subtraction problem types and the range of representations, calculation strategies and tasks used to solve addition and subtraction problems.*

In the FP and IP additive relations points us to the relationship between addition and subtraction. The distinction between FP and IP is the number range and strategies that the learners are expected to work with. A pre-service teacher should be able to understand the following:

Representations	Strategies and Fluencies	Instructional Discourse			Foundational Concepts Standards
<ul style="list-style-type: none"> objects diagrams words (less and more) number track beadstring number line number chart p-p-whole diagram Dienes blocks base-5 and base-10 reps (5-frame, 10-frame, abacus, arithmetic rack) 	<p><u>Fluencies</u></p> <ul style="list-style-type: none"> number facts adding and subtracting 10 to/from any number adding and subtracting tens and hundreds adding a single digit to a decuple (e.g. $30+7=\square$) adding up to a decuple (e.g. $34+\square=40$) subtracting a single digit from a decuple (e.g. $50-6=\square$) subtracting to a decuple (e.g. $78-\square=70$) <p><u>Strategies</u></p> <ul style="list-style-type: none"> counting using known facts (bonds) bridge-through-ten jump strategies (N10) split strategies (1010) compensation 	SAY	DO	WRITE	Aggregates, counting, parts and whole, sets and set relations, number system, relations and fractions, equivalence and order relations, operations as function, computational structures, structure preservation and representations.
		<ul style="list-style-type: none"> • speak about addition as a combination of two sets to produce a combined set, and also as increasing a starting quantity by a certain amount • refer to subtraction as take away and difference: starting with a collection, taking away from it and being left with a smaller collection, and also as making a comparison between two amounts • additive property – the quantity represented by the whole numeral is the sum of the values of the individual digits (e.g. 36 is the sum of 30 and 6) • additive reasoning speaks about how quantities are related in terms of <i>how much more or less</i>. • talk about the inverse 	<ul style="list-style-type: none"> • demonstrate addition as 'forward' jumps on a number track or number line and subtraction as 'backward' jumps • present different types of bare calculations and word problems: join, separate, part-part-whole and compare (the names for the different problem types do not have to be mentioned or explained to learners) • vary the position of the 'unknown', that is, start unknown, change unknown and result unknown 	<ul style="list-style-type: none"> • record addition as 'forward' jumps on a number track or number line and subtraction as 'backward' jumps • calculations can be recorded using number sentences (or the column algorithm) in symbolic form – these can also be recorded as jumps on a number line • flow diagrams and arrow notation can also be used to represent additive reasoning 	

Sub-Standards of the Number sense Knowledge and Practice Standards



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Curriculum Content Sub- Standards – Refers to the teaching curriculum content for Foundation and Intermediate Phase

Understand counting as a fundamental mediating resource used to relate operations on aggregates to the basic arithmetic operations.

Understand the use of additive relations in the teaching of addition and subtraction of natural numbers.

Teachers should know addition and subtraction problem types and the range of representations, calculation strategies and tasks used to solve addition and subtraction problems.

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Pedagogical Sub-Standards

Representations

Refer to contexts used for posing tasks, for supporting learners' reasoning about a task and for classroom discussions.

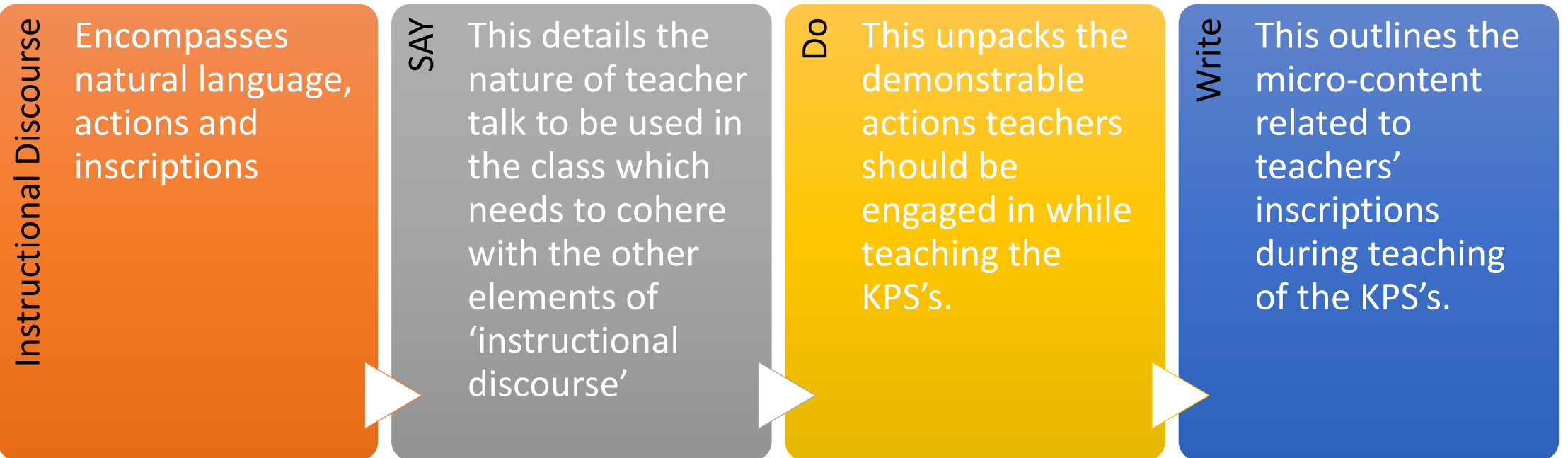
Fluencies

Refers to the 'instant and accurate' manner of responding to a mathematical calculation or procedure

Strategies

Referred to as strategies or methods

Pedagogical Sub-Standards



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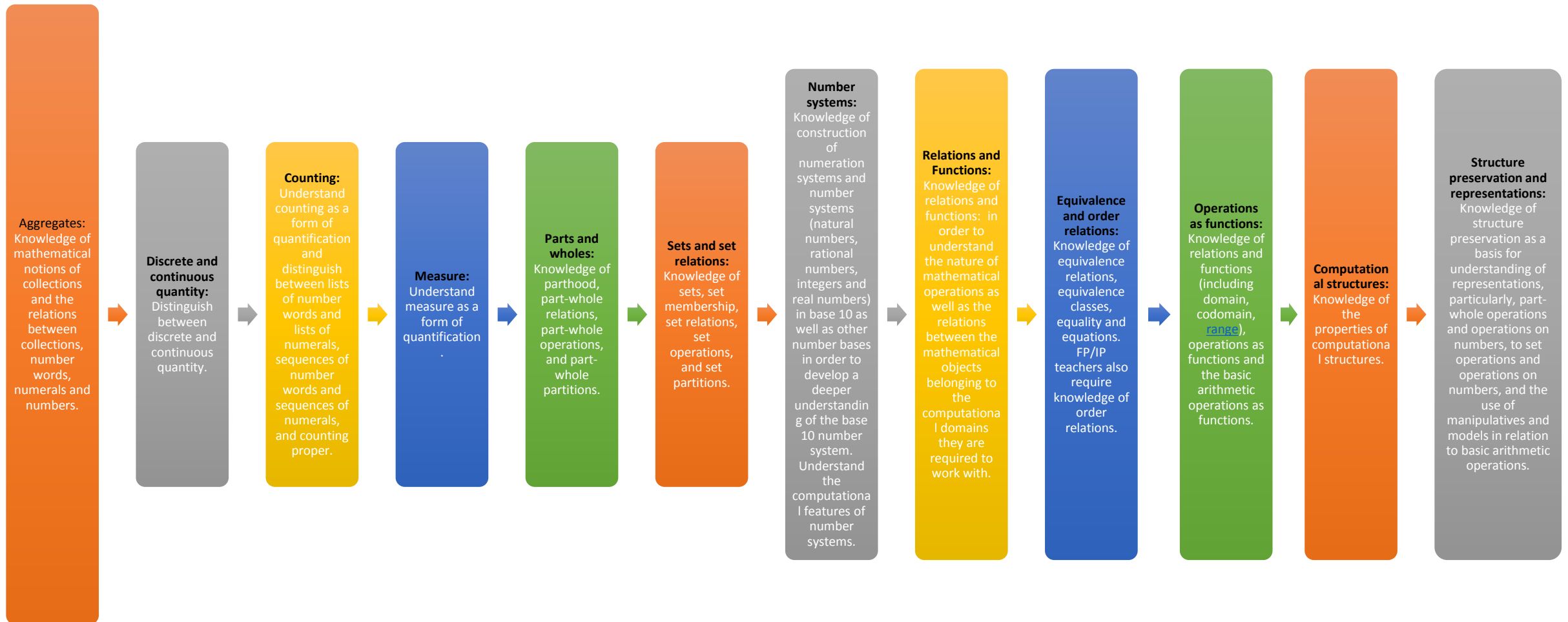
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12 Foundational Concepts Sub-Standards



Foundational Concepts Sub-Standards

- Aggregates, counting, parts and wholes, sets and set relations, number system, relations and fractions, equivalence and order relations, operations as function, computational structures, structure preservation and representations.

Foundational Concepts Sub-Standards

Parts and wholes: Knowledge of parthood, part-whole relations, part-whole operations, and part-whole partitions.

Whole	
Part	Part

10	
8	2

$8+2=10$	$10=8+2$
$2+8=10$	$10=2+8$
$10-2=8$	$8=10-2$
$10-8=2$	$2=10-8$